

Quaternion Algebras References

Quaternionic basics

H. Aslaksen, “Quaternionic determinants,” *Math. Intelligencer* **18** (1996), 57–65.

H.-D. Ebbinghaus *et al.*, *Numbers*, Springer-Verlag, New York, 1990. (See particularly Chapter 3 §1 and Chapter 7.)

T. J. Kaczynski, “Another proof of Wedderburn’s theorem,” *Amer. Math. Monthly* **71** (1964), 652–653. (The most interesting aspect of this proof is the author.)

T. Y. Lam, “Hamilton’s quaternions,” pp. 429–454 of *Handbook of algebra, Vol. 3*, North-Holland, Amsterdam, 2003.

Division rings, the Brauer group

R. K. Dennis and B. Farb, *Noncommutative algebra*, Springer-Verlag, New York, 1993. (The end of the book defines Brauer groups of commutative rings.)

T. Y. Lam, *A first course in noncommutative rings*, 2nd ed., Springer-Verlag, New York, 2001. (Nothing on the Brauer group. Chapter 14 includes simple proofs of Frobenius’ theorem on real division rings and Wedderburn’s theorem on finite division rings. Also a treatment of polynomials with coefficients in a division ring.)

R. S. Pierce, *Associative algebras*, Springer-Verlag, New York, 1982. (Exercise 2 in Section 1.6 discusses $(a, b)_F$ in characteristic 2.)

Quaternion algebras

S. Johansson, <http://www.math.chalmers.se/~sj/forskning.html> (A description of quaternion algebras)

W. Scharlau, *Quadratic and Hermitian forms*, Springer-Verlag, Berlin, 1985. (See Chapter 2 §11 for characteristic $\neq 2$ and Chapter 8 §11 for characteristic 2.)

M.-F. Vignéras, *Arithmétique des algèbres de quaternions*, Springer-Verlag, Berlin, 1980.

Orders

E. Kleinert, “Units of classical orders: a survey,” *Enseign. Math.* **40** (1994), 205–248.

E. Kleinert, *Units in skew fields*, Birkhäuser, Basel, 2000. (Hey’s theorem is discussed at the start.)

D. R. Kohel, “Hecke module structure of quaternions,” pp. 177–195 of *Class field theory – its centenary and prospect*, Math. Soc. Japan, Tokyo, 1998. (See especially section 2, and note discriminants are defined as $\det(\text{Tr}(e_i \bar{e}_j))$, not as $\det(\text{Tr}(e_i e_j))$.)

I. Reiner, *Maximal orders*, Academic Press, London, 1975.

Connections with geometry

I. M. Gel’fand, M. I. Graev, I. I. Pyatetskii-Shapiro, *Representation theory and automorphic functions*, Academic Press, Boston, 1990. (Hey’s theorem is proved on pp. 115–119 for orders of the form $(a, b)_{\mathbf{Z}}$ with $a, b \in \mathbf{Z}^+$.)

S. Johansson, “On fundamental domains of arithmetic Fuchsian groups,” *Math. Comp.* **69** (2000), 339–349. (Includes some examples over real quadratic fields. Uses the Poincaré disk model instead of the upper half-plane model of the hyperbolic plane.)

D. R. Kohel and H. A. Verrill, “Fundamental domains for Shimura curves,” *J. Théor. Nombres Bordeaux* **15** (2003), 205–222. (Illustrates fundamental domains for a maximal order of a few quaternion algebras.)

C. Maclachlan and A. Reid, *The arithmetic of hyperbolic 3-manifolds*, Springer-Verlag, New York, 2003. (The role of quaternion algebras in 3-dimensional geometry, written for geometers.)

T. Miyake, *Modular forms*, Springer-Verlag, Berlin, 1989. (The end of the book discusses the relation between quaternion algebras and modular forms.)

R. Mukundan, “Quaternions: From Classical Mechanics to Computer Graphics, and Beyond,” *Asian Technology Conference in mathematics – ATCM2002*, Melaka, Malaysia, 97–106. Also available at http://www.cosc.canterbury.ac.nz/people/mukundan/atcm02_1.pdf.

J. Silverman, *The Arithmetic of Elliptic Curves*, Springer-Verlag, New York, 1986. (Quaternion algebras are related to supersingular elliptic curves.)

Beyond quaternions

J. C. Baez, “The octonions,” *Bull. Amer. Math. Soc.* **39** (2002), 145–205. (Errata in: *Bull. Amer. Math. Soc.* **42** (2005), 213.)

J. H. Conway and D. A. Smith, *On quaternions and octonions: their geometry, arithmetic, and symmetry*, A. K. Peters, Natick, MA, 2003. (Unique factorization in the Hurwitz order of \mathbf{H} is treated here.)

T. Y. Lam, “A fantasia on quaternions and near-fields,” *Exposition. Math.* **16** (1998), 85–93.