SUMS OF SQUARES: SET 4

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1. Factor 13, 17, and 51 into primes in $\mathbf{Z}[i]$.

2. Factor $2 + 9i$ into primes in $\mathbf{Z}[i]$. (Hint: factor its norm in $\mathbf{Z}$ first.)

3. In $\mathbf{Z}$, the prime factorizations $6 = 2 \cdot 3 = (-2)(-3)$ are essentially the same since the only difference is a sign change. Keeping in mind that $\{\pm 1, \pm i\}$ plays the role in $\mathbf{Z}[i]$ of $\{\pm 1\}$ in $\mathbf{Z}$, show the prime factorizations

$$5 = (1 + 2i)(1 - 2i) = (2 + i)(2 - i).$$

in $\mathbf{Z}[i]$ are essentially the same.

4. Do the factorizations $10 = 2 \cdot 5 = (1 + 3i)(1 - 3i)$ show $\mathbf{Z}[i]$ does not have unique prime factorization?

5. Let p be a prime in $\mathbf{Z}$ and suppose $p|(a^2 + b^2)$ where a and b are integers *without* a common factor. Deduce that p is not prime in $\mathbf{Z}[i]$ by the same kind of argument used to prove Fermat's 2-square theorem from class.