VOLUME WITH $y = \sqrt{x}$ BASE AND SEMICIRCULAR SLICES

The base is the region between $y = \sqrt{x}$, the x-axis, and the line $x = 9$. We make x-slices in the form of semicircles on this base region. A circle of radius r has area πr^2, so a semicircle with radius r has area $\frac{1}{2}\pi r^2$.

Actually, in this picture we naturally see the *diameter* of the semicircles rather than the radius: the diameter is simply the total length of the x-slice segment in the base region. Since the region is bounded by $y = \sqrt{x}$, the diameter of the semicircular x-slice is $\sqrt{x}$, so its radius is $\frac{1}{2}\sqrt{x}$. Therefore the area of the x-slice is

$$A(x) = \frac{1}{2}\pi \left(\frac{1}{2}\sqrt{x}\right)^2 = \frac{\pi}{8}x,$$

so the volume of the solid is

$$\int_0^9 A(x)\,dx = \int_0^9 \frac{\pi}{8}x\,dx = \frac{\pi}{8}\int_0^9 x\,dx = \frac{81\pi}{16}.$$