VOLUME WITH QUARTER UNIT CIRCLE BASE AND SQUARE SLICES

The quarter circle on the base is given by the equation $y = \sqrt{1 - x^2}$ with $0 \le x \le 1$. The square made by the x-slice has area $s(x)^2$, where $s(x)$ is the length of the segment along the quarter circle x-slice. Since $y = \sqrt{1 - x^2}$ along the circle, $s(x) = \sqrt{1 - x^2}$. Therefore the area of the x-slice is

$$A(x) = s(x)^2 = 1 - x^2,$$

so the volume of the solid is

$$\int_0^1 A(x)\,dx = \int_0^1 s(x)^2\,dx = \int_0^1 (1 - x^2)\,dx,$$

which is a polynomial integral whose value you can check is $\frac{2}{3}$.