VOLUME WITH CIRCULAR BASE AND ISOSCELES RIGHT TRIANGLE SLICES

The base is a circle with radius 2 and we place isosceles right triangles across it, where the hypotenuses are lying on the circle and the two equal sides of the triangle are, well, the other two sides. Let's label points on the main diameter of the circle (along which triangles are placed) as x, where $-2 \leq x \leq 2$. Writing $A(x)$ for the area of the isosceles right triangle x-slice, the volume of the solid is

$$\int_{-2}^{2} A(x)\, dx.$$

The easiest aspect of the isosceles triangle on the x-slice to find is its hypotenuse, which is lying on the circle along the x-slice. How long is this side of the triangle? Its two endpoints each lie on the circle, whose equation is $x^2+y^2 = 4$, so the two endpoints on the x-slice have $y = \pm\sqrt{4-x^2}$. Their distance apart is

$$\sqrt{4-x^2} - -\sqrt{4-x^2} = 2\sqrt{4-x^2}.$$

This is the length of the hypotenuse of the isosceles triangle on the x-slice.

What is the area of an isosceles right triangle with hypotenuse h? The two legs of such a triangle (which meet at a 90 degree angle) have length $\frac{h}{\sqrt{2}}$, so the area of the triangle is $\frac{1}{2}\frac{h}{\sqrt{2}}\frac{h}{\sqrt{2}} = \frac{h^2}{4}$. Therefore the triangle on the x-slice, whose hypotenuse has length $2\sqrt{4-x^2}$, has area

$$A(x) = \frac{(2\sqrt{4-x^2})^2}{4} = 4 - x^2.$$

The volume of the solid is

$$\int_{-2}^{2} A(x)\, dx = \int_{-2}^{2} (4-x^2)\, dx.$$

This is a polynomial integral which you can check is $\frac{32}{3}$.