## VOLUME WITH QUARTER UNIT CIRCLE BASE AND SQUARE SLICES

The quarter circle on the base is given by the equation  $y = \sqrt{1 - x^2}$  with  $0 \le x \le 1$ . The square made by the x-slice has area  $s(x)^2$ , where s(x) is the length of the segment along the quarter circle x-slice.



Since  $y = \sqrt{1 - x^2}$  along the circle,  $s(x) = \sqrt{1 - x^2}$ . Therefore the area of the *x*-slice is

$$A(x) = s(x)^2 = 1 - x^2$$

so the volume of the solid is

$$\int_0^1 A(x) \, dx = \int_0^1 s(x)^2 \, dx = \int_0^1 (1 - x^2) \, dx,$$

which is a polynomial integral whose value you can check is  $\frac{2}{3}$ .