## VOLUME WITH QUARTER UNIT CIRCLE BASE AND SQUARE SLICES

The quarter circle on the base is given by the equation $y=\sqrt{1-x^{2}}$ with $0 \leq x \leq 1$. The square made by the $x$-slice has area $s(x)^{2}$, where $s(x)$ is the length of the segment along the quarter circle $x$-slice.


Since $y=\sqrt{1-x^{2}}$ along the circle, $s(x)=\sqrt{1-x^{2}}$. Therefore the area of the $x$-slice is

$$
A(x)=s(x)^{2}=1-x^{2},
$$

so the volume of the solid is

$$
\int_{0}^{1} A(x) d x=\int_{0}^{1} s(x)^{2} d x=\int_{0}^{1}\left(1-x^{2}\right) d x
$$

which is a polynomial integral whose value you can check is $\frac{2}{3}$.

