ETYMOLOGY OF TRIGONOMETRIC FUNCTION NAMES

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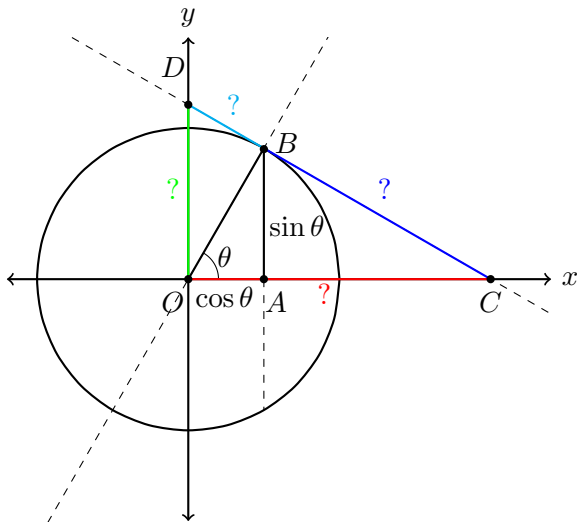
In trigonometry, the name "sine" comes through Latin from a Sanskrit word meaning "chord". In the picture of a unit circle below, $\overline{AB}$ has length $\sin\theta$ and this is half a chord of the circle.

The co-functions are functions of complementary angles: $\cos\theta = \sin(\pi/2 - \theta)$, $\cot\theta = \tan(\pi/2 - \theta)$, and $\csc\theta = \sec(\pi/2 - \theta)$. The names "tangent" and "secant" come from lengths of line segments that are either tangent to the unit circle or cut through a unit circle; a line that cuts through a circle is called a secant line.[1] The figure below will lead to an explanation of these names.



Triangle OAB in the first quadrant of the unit circle is drawn with angle θ at the origin. Its legs, by definition, have lengths $|\overline{OA}| = \cos\theta$ and $|\overline{AB}| = \sin\theta$. The *tangent line* to the circle at the point $B = (\cos\theta, \sin\theta)$ is drawn perpendicularly to the circle and the x and y-axes are *secant lines* of the circle. Mark where these secant lines meet the tangent line as points C and D. How long are the segments $\overline{BC}$, $\overline{OC}$, $\overline{BD}$, and $\overline{OD}$?

Right triangles OAB and OBC are similar, as they have an angle θ. Since $|\overline{OB}| = 1$,

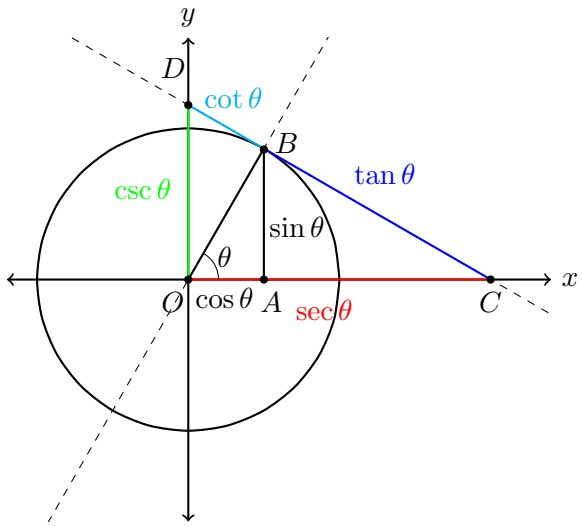
$$\frac{|\overline{BC}|}{|\overline{OB}|} = \frac{|\overline{AB}|}{|\overline{OA}|} \Longrightarrow |\overline{BC}| = \frac{\sin\theta}{\cos\theta} = \tan\theta \text{ and } \frac{|\overline{OC}|}{|\overline{OB}|} = \frac{|\overline{OB}|}{|\overline{OA}|} \Longrightarrow |\overline{OC}| = \frac{1}{\cos\theta} = \sec\theta.$$

The measure of $\angle BOD$ is $\pi/2 - \theta$ and $\angle OBD$ is a right angle, so $\angle ODB$ has measure θ. Therefore the right triangles OBD and OAB have an angle θ, so they are similar. Reasoning as above,

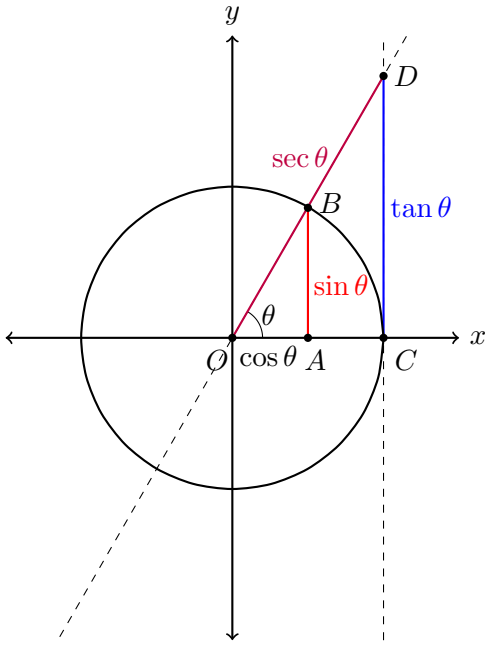
$$\frac{|\overline{BD}|}{|\overline{OB}|} = \frac{|\overline{OA}|}{|\overline{AB}|} \Longrightarrow |\overline{BD}| = \frac{\cos\theta}{\sin\theta} = \cot\theta \text{ and } \frac{|\overline{OD}|}{|\overline{OB}|} = \frac{|\overline{OB}|}{|\overline{AB}|} \Longrightarrow |\overline{OD}| = \frac{1}{\sin\theta} = \csc\theta.$$

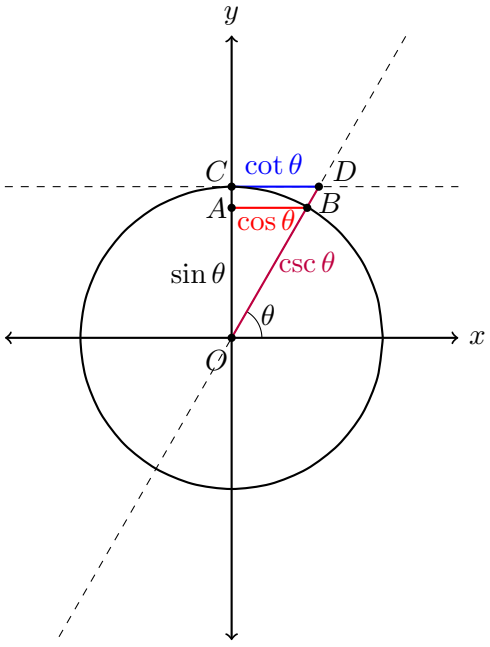
[1] In Latin, *tangere* means "to touch" and *secare* means "to cut". Compare with "section."

The figure below replaces the question marks with the lengths we found, giving geometric interpretations of $\tan\theta$ and $\cot\theta$ as tangent line segment lengths coming from complementary angles, and $\sec\theta$ and $\csc\theta$ as secant line segment lengths coming from complementary angles.



The figures below are an alternate explanation for the tangent and secant function (and co-function) names, using tangent lines at the points $(1,0)$ and $(0,1)$ instead of at the point $(\cos\theta,\sin\theta)$ and the secant line OB instead of the axes. They show $\tan\theta, \sec\theta, \cot\theta$, and $\csc\theta$ are line segment lengths along alternate tangent and secant lines: in the first figure $\tan\theta = |\overline{CD}|$ and $\sec\theta = |\overline{OD}|$, and in the second figure $\cot\theta = |\overline{CD}|$ and $\csc\theta = |\overline{OD}|$.





Historically there were more trigonometric function names (*e.g.*, versin θ for $1-\cos\theta$), but they are now obsolete. Maybe new ones are on the way: see http://www.theonion.com/article/nations-math-teachers-introduce-27-new-trig-functi-33804.