REPRESENTATIONS OF $\text{Aff}(\mathbb{F}_q)$ AND $\text{Heis}(\mathbb{F}_q)$

KEITH CONRAD

For each prime power $q$, we will construct all irreducible representations over $\mathbb{C}$ of the groups $\text{Aff}(\mathbb{F}_q)$ and $\text{Heis}(\mathbb{F}_q)$. To find all of them, there are three parts:

- build as many irreducible representations as the number of conjugacy classes,
- show a representation is irreducible by checking its character has inner product 1 with itself (1-dimensional representations are automatically irreducible),
- show irreducible representations are nonisomorphic by checking their characters are different.

1. Representations of $\text{Aff}(\mathbb{F}_q)$

Let $\mathbb{F}$ be a field. In $\text{Aff}(\mathbb{F})$, the group law is

$$\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a' & b' \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} aa' & ab' + b \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/a & -b/a \\ 0 & 1 \end{pmatrix}.$$

Two subgroups of $\text{Aff}(\mathbb{F})$ are

$$\left\{ \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} : a \neq 0 \right\}, \quad \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{F} \right\},$$

which are isomorphic to $\mathbb{F}^\times$ and $\mathbb{F}$ as groups. Matrices in $\text{Aff}(\mathbb{F})$ decompose into a product of elements in these subgroups as

$$\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}.$$

The product in the other order doesn’t work in the same way:

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & ab \\ 0 & 1 \end{pmatrix}.$$

Conjugation in $\text{Aff}(\mathbb{F})$ is

$$(1.1) \quad \begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} a & bx - y(a - 1) \\ 0 & 1 \end{pmatrix}.$$

By (1.1), $\text{Aff}(\mathbb{F})$ has trivial center unless $\mathbb{F} = \mathbb{F}_2$, in which case $\text{Aff}(\mathbb{F})$ is abelian.\footnote{Use $x = y = 1$ to see a matrix in the center of $\text{Aff}(\mathbb{F})$ has $a = 1$. If $\mathbb{F} \neq \mathbb{F}_2$, so $\mathbb{F}^\times \neq \{1\}$, then use $y = 0$ and $x \neq 1$ to see a matrix in the center of $\text{Aff}(\mathbb{F})$ has $b = 0$.}

Here are the conjugacy classes in $\text{Aff}(\mathbb{F})$:

- the identity matrix $$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\},$$
- the set $$\left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{F}^\times \right\},$$
for each \( a \in F \) with \( a \neq 0 \) and \( a \neq 1 \), the set
\[
\left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : b \in F \right\}.
\]
So in \( \text{Aff}(F_q) \) there are a total of \( 1 + 1 + (q - 2) = q \) conjugacy classes and thus there are \( q \) irreducible representations of \( \text{Aff}(F_q) \) over \( \mathbb{C} \).

**One-dimensional representations:** Since the upper left entry in \( \text{Aff}(F_q) \) behaves multiplicatively in the group law, for each homomorphism \( \chi : F_q^\times \to \mathbb{C}^\times \) we get a one-dimensional representation \( \text{Aff}(F_q) \to \mathbb{C}^\times \) by
\[
\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \mapsto \chi(a).
\]
Since \( F_q^\times \) is cyclic of order \( q - 1 \), there are \( q - 1 \) such \( \chi \), so we get \( q - 1 \) one-dimensional representations of \( \text{Aff}(F_q) \).

**Remaining irreducible representation:** From the count of conjugacy classes there is one more irreducible representation of \( \text{Aff}(F_q) \). Letting \( d \) denote its degree, from \( q - 1 + d^2 = |\text{Aff}(F_q)| = q(q - 1) \) we get \( d = q - 1 \), so we seek a \( (q - 1) \)-dimensional representation.

Consider the complex vector space \( V \) of functions \( f : F_q \to \mathbb{C} \). This is \( q \)-dimensional. Let each \( g \in \text{Aff}(F_q) \) act on \( V \) as a linear change of variables using \( g^{-1} : (\rho_V(g)f)(x) = f(g^{-1}x) \). We need \( g^{-1} \) rather than \( g \) in the formula to get \( \rho_V(gh) = \rho_V(g)\rho_V(h) \). Explicitly,
\[
(\rho_V(\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix})f)(x) = f \left( \frac{1}{a}x - \frac{b}{a} \right).
\]
The constant functions in \( V \) form a one-dimensional subspace on which \( \text{Aff}(F_q) \) acts trivially. Another \( \text{Aff}(F_q) \)-stable subspace of \( V \) is
\[
W = \left\{ f \in V : \sum_{x \in F_q} f(x) = 0 \right\}
\]
and \( \rho_V = \rho_W \oplus 1 \), where \( \rho_W \) is the restriction of \( \rho_V \) to \( W \). The dimension of \( W \) is \( q - 1 \).

To show \( W \) is irreducible, we compute its character from that of \( V \): \( \chi_V = \chi_W + 1 \). A basis of \( V \) is the \( q \) delta-functions \( \delta_t : F_q \to \mathbb{C} \) for \( t \in F_q \), where \( \delta_t(x) = 0 \) for \( x \neq t \) and \( \delta_t(t) = 1 \). Since \( \rho_V(\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix})\delta_t = \delta_{at+b} \), the matrix for \( \rho_V(\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}) \) with respect to the delta-basis of \( V \) is a permutation matrix that describes how \( t \mapsto at + b \) permutes \( F_q \). Thus
\[
\chi_V \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} = |\{ t \in F_q : at + b = t \text{ in } F_q \}| = \begin{cases} 1, & \text{if } a \neq 1, \\ q, & \text{if } a = 1 \text{ and } b = 0, \\ 0, & \text{if } a = 1 \text{ and } b \neq 0. \end{cases}
\]
Therefore
\[
\chi_W \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} = \chi_V \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} - 1 = \begin{cases} 0, & \text{if } a \neq 1, \\ q - 1, & \text{if } a = 1 \text{ and } b = 0, \\ -1, & \text{if } a = 1 \text{ and } b \neq 0. \end{cases}
\]
The inner product of \( \chi_W \) with itself is
\[
\frac{1}{q(q - 1)} \sum_g \chi_W(g)\overline{\chi_W(g)} = \frac{1}{q(q - 1)}((q - 1)^2 + (q - 1)(-1)^2) = \frac{(q - 1)^2 + (q - 1)}{q(q - 1)} = 1,
\]
so $W$ is irreducible. It is a new irreducible representation since it’s not 1-dimensional, except if $q = 2$, in which case $\rho_W$ is nontrivial while the single one-dimensional representation constructed earlier ($q - 1 = 1$ if $q = 2$) is trivial (note $\text{Aff}(F_2) \cong F_2$).

2. Representations of $\text{Heis}(F_q)$

For a field $F$, the group law in the Heisenberg group $\text{Heis}(F)$ is

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a' & b' \\ 0 & 1 & c' \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+a' & b+b' + ac \\ 0 & 1 & c+c' \\ 0 & 0 & 1 \end{pmatrix},$$

with inverse formula

$$\begin{pmatrix} 1 & a \\ 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -a \\ 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}. $$

Three subgroups of $\text{Heis}(F)$ are

$$\left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a \in F \right\}, \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : b \in F \right\}, \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : c \in F \right\},$$

which are each isomorphic as groups to the additive group of $F$. Note the subset

$$\left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, c \in F \right\}$$

is not a subgroup of $\text{Heis}(F)$ since it’s not closed under multiplication.

Each matrix in $\text{Heis}(F)$ is a product of matrices in the three subgroups above:

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

If we multiply these three matrices in the reverse order,

$$\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a & b + ac \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}.$$

Conjugation in $\text{Heis}(F)$ is described by the formula

$$\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & a & b - az + cx \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}.$$

In particular, the center of $\text{Heis}(F)$ is

$$\left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : b \in F \right\}.$$

Using the conjugation formula (2.3), we get the conjugacy classes in $\text{Heis}(F)$:
• for each $b \in F$, the single matrix
  \[
  \begin{pmatrix}
  1 & 0 & b \\
  0 & 1 & 0 \\
  0 & 0 & 1 
  \end{pmatrix}
  \]

• for each pair $(a, c) \in F^2 - \{(0,0)\}$, the set
  \[
  \left\{ \begin{pmatrix}
  1 & a & b \\
  0 & 1 & c \\
  0 & 0 & 1 
  \end{pmatrix} : b \in F \right\}.
  \]

When $F = F_q$, there are $q + (q^2 - 1) = q^2 + q - 1$ conjugacy classes in $\text{Heis}(F_q)$, so this group has $q^2 + q - 1$ irreducible representations over $C$.

**One-dimensional representations:** Fix a nontrivial homomorphism $\psi : F_q \to C^\times$. An example is $\psi(a) = e^{2\pi i \text{Tr}(a/p)}$, where $F_q$ has characteristic $p$ and $\text{Tr}_{F_q/F_p} : F_q \to F_p$ is the trace map. (If $q = p$ then $\psi : F_p \to C^\times$ by $\psi(a) = e^{2\pi i a/p}$.) Since the $a$ and $c$ terms of a matrix in $\text{Heis}(F_q)$ each combine additively under multiplication in $\text{Heis}(F_q)$, for each $(x, y) \in F_q^2$ there is a 1-dimensional representation $\psi_{x,y} : \text{Heis}(F_q) \to C^\times$ given by

\[
(2.5) \quad \psi_{x,y} \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \psi(xa + yc).
\]

Check as an exercise that from $\psi : F_q \to C^\times$ being nontrivial, if $\psi_{x,y} = \psi_{x',y'}$ then $(x, y) = (x', y')$ in $F_q^2$, so \{ $\psi_{x,y} : (x, y) \in F_q^2$ \} is $q^2$ irreducible representations of degree 1. (Hint: first show that if $x \neq 0$ or $y \neq 0$ then $\psi_{x,y}$ is nontrivial, i.e., is not identically 1.)

**Remaining irreducible representations:** The number of remaining irreducible representations is \((q^2 + q - 1) - q^2 = q - 1\). Their degrees satisfy $q^2 + \sum d_i^2 = |\text{Heis}(F_q)| = q^3$, so $\sum d_i^2 = q^3 - q^2 = (q - 1)q^2$. We will find $q - 1$ irreducible representations of degree $q$.

Let $V$, as before, be the $q$-dimensional vector space of functions $f : F_q \to C$. We will define three actions of $F_q$ on $V$, and composing them in a suitable order will give an action of the group $\text{Heis}(F_q)$ on $V$. Fix a nontrivial homomorphism $\psi : F_q \to C^\times$. For $a, b, c \in F_q$, define the linear maps $\sigma_a : V \to V$, $\tau_b : V \to V$, and $\varphi_c : V \to V$ by

\[
(\sigma_a f)(x) = f(x + a), \quad (\tau_b f)(x) = \psi(b) f(x), \quad (\varphi_c f)(x) = \psi(cx) f(x).
\]

For $t \in F_q$, note $\tau_t$ multiplies each function in $V$ by the number $\psi(t)$ while $\varphi_t$ multiplies each function in $V$ by the function $\psi(tx)$.

Check $\sigma_a \circ \sigma_{a'} = \sigma_{a+a'}$, $\tau_b \circ \tau_{b'} = \tau_{b+b'}$, and $\varphi_c \circ \varphi_{c'} = \varphi_{c+c'}$ as functions $V \to V$, so $a \mapsto \sigma_a$, $b \mapsto \tau_b$, $c \mapsto \varphi_c$ are actions of $F_q$ on $V$ by linear maps. Check $\tau_t$ commutes with both $\sigma_a$ and $\varphi_c$, while $\sigma_a$ and $\varphi_c$ commute with each other up to scaling by a root of unity:

\[
(2.6) \quad \sigma_a \circ \tau_b = \tau_b \circ \sigma_a, \quad \varphi_c \circ \tau_b = \tau_b \circ \varphi_c, \quad \sigma_a \circ \varphi_c = \psi(ac) \varphi_c \circ \sigma_a = \tau_{ac} \circ \varphi_c \circ \sigma_a.
\]

Using (2.6), the 3-fold composites $\sigma_a \circ \tau_b \circ \varphi_c$ compose as follows:

\[
(\sigma_a \circ \tau_b \circ \varphi_c) \circ (\sigma_{a'} \circ \tau_{b'} \circ \varphi_{c'}) = (\sigma_a \circ \tau_b \circ \varphi_c) \circ (\sigma_{a'} \circ \tau_{b'} \circ \varphi_{c'})
= \sigma_a \circ \tau_b \circ \tau_{a'c} \circ \sigma_{a'} \circ \varphi_c \circ \varphi_{c'}
= \sigma_a \circ \tau_{a'b+c} \circ \sigma_{a'} \circ \tau_{b'} \circ \varphi_c \circ \varphi_{c'}
= \sigma_a \circ \sigma_{a'} \circ \tau_{a'b-c} \circ \tau_{b'} \circ \varphi_{c} \circ \varphi_{c'}
= \sigma_{a+a'} \circ \tau_{b+b'} \circ \varphi_{c+c'}.
\]
This is almost like the way matrices in Heis($\mathbb{F}_q$) multiply in (2.1), and it would match matrix multiplication if $\tau_{b+b'}-a'$ were $\tau_{b+b'+a'}$. Matrices in Heis($\mathbb{F}_q$) decompose in (2.2) using reverse alphabetical order, which suggests looking at $\varphi_c \circ \tau_b \circ \sigma_a$ instead. By calculations as above,

\[
\begin{align*}
\varphi_c \circ \tau_b \circ \sigma_a \circ (\varphi_c' \circ \tau_{b'} \circ \sigma_{a'}) &= \varphi_c \circ \tau_b \circ (\sigma_a \circ \varphi_c') \circ \tau_{b'} \circ \sigma_{a'} \\
&= \varphi_c \circ \tau_b \circ (\tau_{ac'} \circ \varphi_c' \circ \sigma_a) \circ \tau_{b'} \circ \sigma_{a'} \\
&= \varphi_c \circ \tau_{b+ac'} \circ \varphi_c' \circ \tau_{b'} \circ \sigma_a \circ \sigma_{a'} \\
&= \varphi_c \circ \varphi_{c'} \circ \tau_{b+ac'} \circ \tau_{b'} \circ \sigma_a \circ \sigma_{a'} \\
&= \varphi_{c+c'} \circ \tau_{b+b'+ac'} \circ \sigma_{a+a'}.
\end{align*}
\]

This is exactly how matrices in Heis($\mathbb{F}_q$) multiply, so we can define a $q$-dimensional representation $\rho_\psi$ of Heis($\mathbb{F}_q$) on $V$ by

\[
g = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mapsto \rho_\psi(g) := \varphi_c \circ \tau_b \circ \sigma_a.
\]

As a formula, for $f \in V$

\[
(\rho_\psi(g)f)(x) = (\varphi_c \tau_b \sigma_a f)(x) = \psi(cx)(\tau_b \sigma_a f)(x) = \psi(cx)\psi(b)f(x+a) = \psi(cx+b)f(x+a).
\]

Let’s compute the character of $\rho_\psi$. Using the basis $\{\delta_t : t \in \mathbb{F}_q\}$ of $V$, we have

\[
g = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mapsto (\rho_\psi(g)\delta_t)(x) = \psi(cx+b)\delta_t(x+a) = \begin{cases} \psi(c(t-a)+b), & \text{if } x = t-a, \\ 0, & \text{if } x \neq t-a, \end{cases}
\]

so

\[
\rho_\psi(g)\delta_t = \psi(c(t-a)+b)\delta_{t-a}.
\]

- If $a \neq 0$, then the matrix of $\rho_\psi(g)$ with respect to the delta-basis has 0’s on the main diagonal, so $\tr(\rho_\psi(g)) = 0$.
- If $a = 0$, then the matrix of $\rho_\psi(g)$ with respect to the delta-basis has diagonal entries $\psi(ct+b)$, so $\tr(\rho_\psi(g)) = \sum_{t \in \mathbb{F}_q} \psi(ct+b)$. If $c \neq 0$ then this sum equals $\sum_{x \in \mathbb{F}_q} \psi(x)$, which is 0 since $\psi$ is nontrivial, and if $c = 0$ then this sum is $q\psi(b)$.

Thus the character $\chi_\psi$ of $\rho_\psi$ is

\[
\chi_\psi \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \begin{cases} q\psi(b), & \text{if } a = c = 0, \\ 0, & \text{otherwise}, \end{cases}
\]

so for each nontrivial homomorphism $\psi : \mathbb{F}_q \to \mathbb{C}^\times$, the inner product of $\chi_\psi$ with itself is

\[
\frac{1}{q^3} \sum_{g} \chi_\psi(g)\overline{\chi_\psi(g)} = \frac{1}{q^3} \sum_{b \in \mathbb{F}_q} q\psi(b)\overline{q\psi(b)} = \frac{1}{q} \sum_{b \in \mathbb{F}_q} 1 = 1.
\]

There are $q-1$ nontrivial homomorphisms $\psi : \mathbb{F}_q \to \mathbb{C}^\times$, and to each is a $q$-dimensional irreducible representation $\rho_\psi$ of Heis($\mathbb{F}_q$). To prove $\rho_\psi$’s for different nontrivial $\psi$ are non-isomorphic, we show their characters $\chi_\psi$ are distinct. This is a consequence of the formula

\[
\chi_\psi \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = q\psi(b),
\]

\[
\chi_\psi \begin{pmatrix} 1 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} 
\]
which is a special case of (2.7). It shows we can recover each nontrivial \( \psi \) from the character \( \chi_\psi \) of the \( q \)-dimensional representation \( \rho_\psi \) of \( \text{Heis}(F_q) \). (Alternatively, check for different nontrivial homomorphisms \( \psi_1, \psi_2 : F_q \to \mathbb{C}^\times \) that the inner product of the characters of \( \rho_{\psi_1} \) and \( \rho_{\psi_2} \) is 0, so for different nontrivial \( \psi \)'s the representations \( \rho_\psi \) are nonisomorphic.)

The center of \( \text{Heis}(F_q) \), by (2.4), is

\[
Z_q := \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : b \in F_q \right\} \cong F_q,
\]

so (2.8) tells us that each \( q \)-dimensional irreducible character \( \chi_\psi \) of \( \text{Heis}(F_q) \) is determined by its restriction to \( Z_q \), since \( \chi_\psi \) on \( Z_q \) is enough information to determine \( \psi \) on \( F_q \), which is all we need to compute \( \chi_\psi \) on \( \text{Heis}(F_q) \) in (2.7). In terms of representations rather than characters, the definition of \( \rho_\psi \) shows that \( \rho_\psi \) on \( Z_q \) is described by

\[
\rho_\psi \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \tau_b : f \mapsto \psi(b) f
\]

for all functions \( f \in V \), so \( \rho_\psi \) restricted to \( Z_q \) is enough information to recover \( \psi \) on \( F_q \).

Each \( q \)-dimensional irreducible representation \( \rho_\psi \) of \( \text{Heis}(F_q) \) is nontrivial on \( Z_q \). The other irreducible representations of \( \text{Heis}(F_q) \) are 1-dimensional, and they are trivial on \( Z_q \) by (2.5).\(^2\) Thus the irreducible representations of \( \text{Heis}(F_q) \) that are nontrivial on \( Z_q \) are in bijection with the nontrivial characters of \( Z_q \) by \( \rho_{\psi} \mapsto \psi \); this is an analogue of the Stone-von Neumann theorem in mathematical physics about representations of \( \text{Heis}(\mathbb{R}) \).\(^3\)

More on the irreducible representations of \( \text{Aff}(F_q) \) and \( \text{Heis}(F_q) \) is in [1, Chap. 16–18].

References


---

\(^2\) The commutator subgroup of \( \text{Heis}(F_q) \) is its center, so that's another reason 1-dimensional representations of \( \text{Heis}(F_q) \) are trivial on the center.

\(^3\) See https://www-users.cse.umn.edu/~garrett/m/repns/notes_2014-15/svn_theorem.pdf.